

# Inherent Error Analysis of Slope Variable with IDW Method

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**Abstract**— This paper deals with inherent error analysis of slope variable due to the IDW interpolation method and its various power values. In the first section it shows slope analysis method and algorithm. In the second section it creates a DEM model from various measured and erroneous elevated points which becomes input to calculate slope of interpolated DEMs separately then error is calculated by calculating difference of the slope for measured and erroneous slope. It explores error analysis of slope with its practical implementation in ArcGIS. In the last section it explains the comparison of errors for various power of IDW method. This paper demonstrates that slope error increases with the increment in the inverse power of distance in IDW method.

**Keywords**— 3D GIS; Slope; Error analysis, IDW, Interpolation.

## I. INTRODUCTION

In last decades GIS technology is changing drastically over the world. It permits to query, interpret identify and visualize spatial and non-spatial data in various ways that expose trends and patterns in the form of maps, graphs and reports etc.

Some specific issues of 3D visualization with in 3D GIS are to explore for example, to visualize 3D spatial analysis result some tools are required to effortlessly explore and navigate real time models [1]

Slope is defined by a plane tangent to a surface at a specific point and is specified in terms of the maximum rate of change of altitude (slope) and the compass direction associated with the maximum (aspect). Slope, therefore, also has a local neighborhood over which it is computed, usually a three by three region. The maximum slope is familiar to skiers as the fall line, and the aspect is the direction in which the fall line trends. When slope is zero, the terrain is flat and the aspect is undefined. Slope is computed by solving a best fit surface through the points in the neighborhood and by measuring the change in elevation per unit distance in this neighborhood and the direction. These values can be assigned as data to a new grid [2] [3]

Inverse Distance Weighted (IDW) Interpolation is one of the most commonly used spatial interpolation method. This method is comparatively easy and fast to compute interpolation than other interpolation methods. This method assumes that attributes of unknown points are weighted average of known values within its neighboring points. Weights of the unknown points are inversely

proportion to the power of distances between the unknown and known points [4].

There are many types of errors, which affect slope i.e. error in source data, data entry error equipment error, error in algorithm used in interpolation etc. Here in this paper we are considering the inherent errors in the data.

M.A. Azpura et al. (2010) [5] compares various interpolation techniques with spatial data but they didn't compare the effect of various inverse powers. Qiming Zhou and Xuejun 2004 [6] explained various errors noticeable errors due to errors in data, inherent in data structures, and created by algorithms in slope derived from grid DEM. So what happens if the slope is calculated from erroneous data. This paper analyses the comparison of errors in slope due to different powers of inverse distance weighted interpolation.

## II. PURPOSE AND OBJECTIVES

M.A. Azpura et al. (2010) [5] compares various interpolation techniques with spatial data but they didn't compare the effect of various inverse powers. Qiming Zhou and Xuejun 2004 [6] explained various errors noticeable errors due to errors in data, inherent in data structure, and created by algorithms in slope derived from grid DEM. So what happens if the slope is calculated from erroneous data. This paper analyses the comparison of errors in slope due to different powers of inverse distance weighted interpolation.

This study focuses on the IDW method and comparative study of various inverse powers in the interpolation of spatial data and its impacts on the slope. The overall purpose is to better understand the IDW method and minimize error impact on the slope.

## III. LITERATURE REVIEW

George Y. Lu et al. 2008, worked on an adaptive IDW interpolation technique [4].

According to Qiming Zhou and Xuejun 2004 [5], identifies noticeable errors due to errors in data, inherent in data structure, and created by algorithms in slope derived from grid DEM.

R. Hickey 2000 [6], described erosion modeling for calculating the cumulative downhill slope length. Along with it, methods for calculating slope and aspect were also defined.

E. M. Masaad and S. M. Moneim (2012) [7] explained suitable design of road pattern for Kosti town based on TIN Analysis.

N. Bhargava, R.Bhargava and P.S. Tanwar (2013) [3] creates the TIN Model from mass points given in GML form.

M.A. Azpura et al. (2010) [8] compared various spatial interpolation methods i.e. spline, IDW and Krigging methods for estimation of average electromagnetic field magnitude.

Many researchers Like Chu, Tsai, O'Callagan, Flemming, Hoffer etc. defined Mathematical model for slope and aspect on the basis of quantitative analysis, trend analysis, Vector based analysis and fast Fourier transform based analysis.

Other researchers Like X. Hanjianga 2008 [9], E. R. Vivoni et al. 2004 [2], G. E. Tucker et al. 2001 [10], R.Pajrola et al. 2002 [11] explained the 3D TIN models.

IV. METHODOLOGY

A. Slope

Slope is an attribute which is used to define surface and it involves gradient and aspect in it.

Gradient is the first vertical derivative of altitude and it is rate of change in its magnitude over distance. Gradient of the surface is known as slope. The slope or gradient of a surface describes its steepness or incline [5].

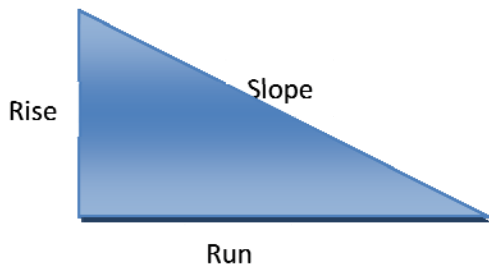


Fig. 1. Slope

Slope can be calculated by interpolating the surface and it is defined as

$$Slope S = \arctan(\sqrt{f_x^2 + f_y^2}) \dots \dots \dots (1)$$

$$S = \tan^{-1} \left( \sqrt{\left(\frac{dZ_x}{dx}\right)^2 + \left(\frac{dZ_y}{dy}\right)^2} \right) \dots \dots \dots (2)$$

Where

$$dZ_x = Z_3 - Z_2, \\ f_x = \frac{dZ_x}{dx}, f_y = \frac{dZ_y}{dy}$$

B. Interpolation

Interpolation is required to identify the heights for the unknown points. There are many interpolation methods which are used to interpolate unknown values from given points.

C. IDW Interpolation

IDW method is based on the supposition that the neighboring point contributes more in interpolation than the farther one. IDW method states that impact of known data point is inversely proportion to the distance from unknown to known data points [8].

Interpolated value in IDW method is defined as

$$w(x) = \begin{cases} \left( \frac{\sum_{i=1}^n \frac{w_i}{d(x, x_i)^p}}{\sum_{i=1}^n \frac{1}{d(x, x_i)^p}} \right) \dots \dots \dots \text{if } [d(x, x_i) \neq 0] \\ w_i \dots \dots \dots \text{if } [d(x, x_i) = 0] \dots \dots (3) \end{cases}$$

where,

- w(x) is the point to be interpolate
- w<sub>i</sub> =weight at point x<sub>i</sub>
- d(x,x<sub>i</sub>) distance from x to x<sub>i</sub>
- p= inverse distance power

IDW interpolation for height

$$h(x, y) = \begin{cases} \left( \frac{\sum_{i=1}^n \frac{h_i}{d((x, y), (x_i, y_i))^p}}{\sum_{i=1}^n \frac{1}{d((x, y), (x_i, y_i))^p}} \right) \dots \dots \dots \text{if } [d(x, y), d(x_i, y_i) \neq 0] \\ h(x, y) = h_i \dots \dots \dots \text{if } [d(x, y), d(x_i, y_i) = 0] \dots \dots (4) \end{cases}$$

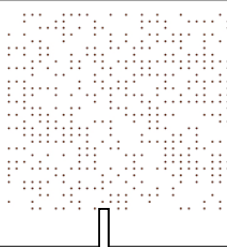
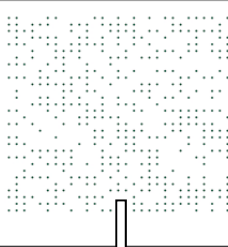
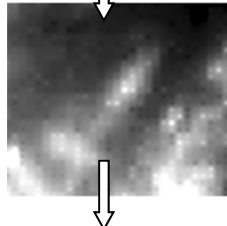
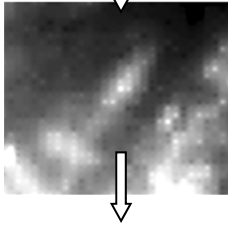
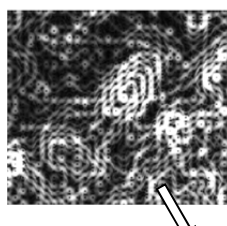
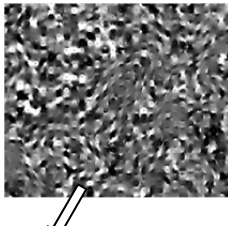
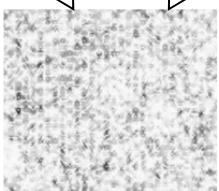
where

- h(x,y) is the height at x,y
- h<sub>i</sub> =height at point (x<sub>i</sub>,y<sub>i</sub>)
- d((x,y),(x<sub>i</sub>,y<sub>i</sub>))=distance from (x,y) to (x<sub>i</sub>,y<sub>i</sub>)
- p= power

$$d((x,y),(x_i,y_i)) = \sqrt{((x-x_i)^2 + (y-y_i)^2)}$$

$$h(x, y) = \begin{cases} \left( \frac{\sum_{i=1}^n \frac{h_i}{\left(\sqrt{((x-x_i)^2 + (y-y_i)^2)}\right)^p}}{\sum_{i=1}^n \frac{1}{\left(\sqrt{((x-x_i)^2 + (y-y_i)^2)}\right)^p}} \right) \dots \dots \dots \text{distance} \neq 0 \\ h(x, y) = h_i \dots \dots \dots \text{if } [d(x, y), d(x_i, y_i) = 0] \dots \dots (5) \end{cases}$$

TABLE I. SLOPE ERROR ANALYSIS PROCESS DUE TO IDW WITH INVERSE OF POWER

S · N	Process	Without Error	Erroneous
1	Elevation Data		
2	IDW Interpolation		
3	Slope		
4	Difference (Correct Slope - Erroneous Slope)		

D. Error

An 'error' is a deviation from accuracy or correctness.

$$\text{Error} = \text{actual data} - \text{experimental data}$$

To make a fair comparison between erroneous data and true data, some error has been inserted into the true data so that correct results are obtained.

V. SLOPE AND IDW IN ARCGIS

ArcGIS provides spatial analyst and Geo-statistical analyst. These Toolsets are useful in calculating and analyzing Interpolation and Slope.

VI. ALGORITHM

Input: Measured Elevation data and call it Correct\_Ele\_data,

Step 1. Put some randomized error in the elevation data and call it "Err\_Ele\_data",

Step 2. Repeat step 2(a) to 2(e) till  $p < n$  otherwise go to step 4

- a.  $\text{IDW\_corr\_Ele} = \text{Interpolate Correct\_Ele\_data Data With IDW interpolation with power } p$  and name it " $\text{IDW\_Corr\_Ele}_p$ "
- b.  $\text{IDW\_Err\_Ele} = \text{Interpolate Err\_Ele\_data Data With IDW interpolation with power } p$  and name it " $\text{IDW\_Err\_Ele}_p$ "
- c.  $\text{Slope\_Corr\_IDW}_p = \text{Create slope raster from interpolated Correct Elevation raster data "Slope\_Corr\_IDW}_p$ "
- d.  $\text{Slope\_Erroneous\_IDW}_p = \text{Create slope raster from interpolated Correct Elevation raster data and call it "Slope\_Error\_IDW}_p$ "
- i.  $\text{Diff\_Slope}_p = \text{Slope\_Corr\_IDW}_p - \text{Slope\_Erroneous\_IDW}_p$  and call it " $\text{Diff\_Slope}_p$ "

Step 3. Goto step 2

Step 4. Calculate the error

Step 5. Plot the graph

The step by step explanation of algorithm is shown with images in the Table 1. In this process elevation data are used to create digital elevation model in raster format using IDW method for both erroneous data and error free data. In the next step slope analysis is carried out from the previous step's result and in the last step difference of each pixel is calculated which is shown in the figure of Table 1. This process is repeated for each inverse power values of the IDW method.

VII. RESULTS AND DISCUSSION

Various Inverse powers in IDW method for the Erroneous data and data without error are used to get the DEM of the surface, which is used to calculate the slope.

Comparison of various slope surfaces obtained from various inverse distance power of IDW method for erroneous and without error data are shown in table 2. It is almost clear from the images of table 2 that inverse power 2 gives a better slope value than the higher inverse power values. As soon as the inverse power values increases, error in slope analysis increases and the smooth curve changes into rough zigzag slope. It shows that it is better to use IDW with inverse power 2 than the higher inverse power values.

Error analysis graph for with respect to various IDW's distance power 2, 3, 4 are given in fig. 2, fig.3, fig. 4 respectively.

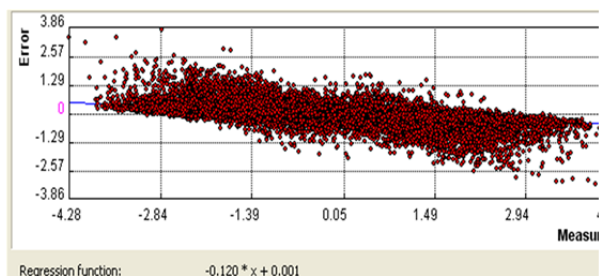
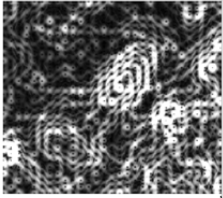
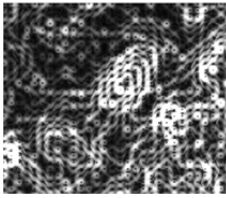
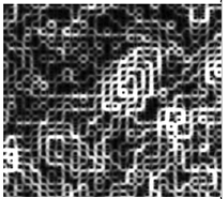
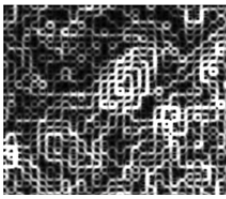


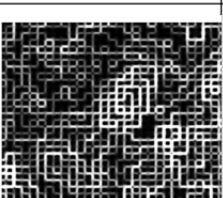
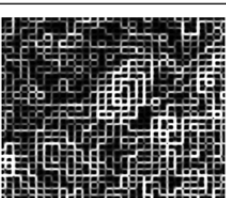
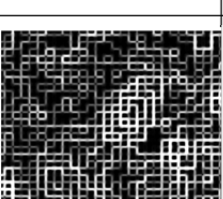
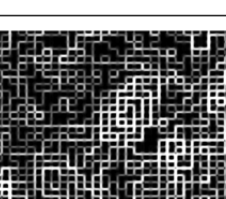


Fig. 2. Slope Error graph with IDW's power 2

TABLE 2. SLOPE WITH AND WITHOUT ERROR (COMPARATIVE STUDY OF IDW'S VARIOUS POWER)

S.No	Inv. Distance Power	Without Error	Erroneous
1	2		
2	3		
3	4		
4	5		
5	6		

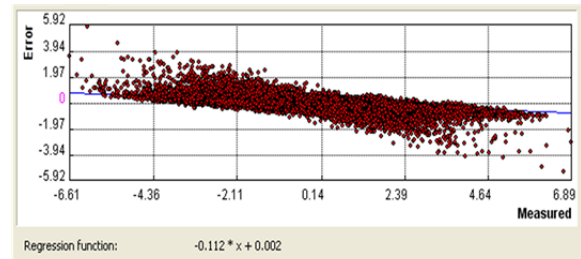


Fig. 4. Slope Error graph with IDW's power 3

Comparison of The IDW Interpolation method with different power parameters in the error analysis of slope is specified in Table 3, and their graphical representation is shown in fig. 4. From this experiment it is clear that as inverse power to the distance increases, range of error also increases. It is observed that for simple and best results IDW interpolation method with inverse power 2 is more suitable than other inverse powers.

TABLE 3. ERROR RANGE FOR VARIOUS INVERSE DISTANCE POWER

S.No	Inverse Distance Power	Min Error	Max Error
1	2	-3.86	3.86
2	3	-5.92	5.92
3	4	-6.77	6.77
4	5	-7.42	7.42
5	6	-8.32	-8.32

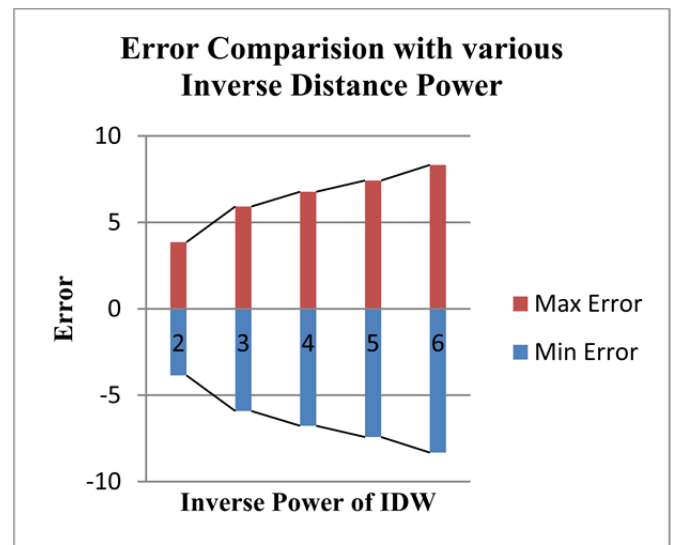


Fig. 5. Slope Error graph with IDW's power

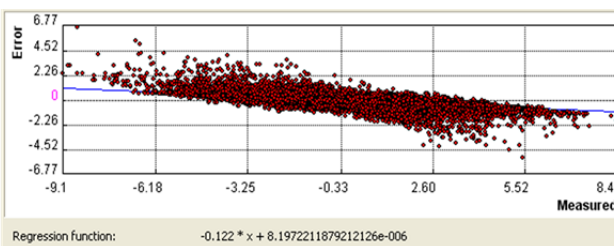


Fig. 3. Slope Error graph with IDW's power 4

### VIII. CONCLUSION

In this study, the slope calculation from raw elevation data of the site is presented. This paper focused on the algorithm generation for error analysis of slope with respect to various inverse distance power values in the IDW method.

The experimental results of practical solutions for creating error analysis of slope using IDW. The comparative study of slope error analysis with various

inverse power values are calculated to find less error prone IDW power.

The usefulness of slope analysis in the real world is in the Environment and climate. Proper slope will protect the area from hot, winds or from cold can be good for agriculture.

Error free or less error prone slope analysis is required in almost every application areas like water flow analysis, vegetation analysis, terrain stability assessment, snow avalanche risk mapping.

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